THE NP-COMPLETENESS OF THE OPTIMAL DISK CYLINDER ARRANGEMENT PROBLEM

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The Optimal Disk Cylinder Arrangement problem is that of rearranging and remapping cylinders on a moving-head disk to minimize expected seek time. The position of the head is modeled as a Markov chain, with conditional transition (head movement) probabilities p_{ij} for any two cylinders i and j. The unconditional transition probabilities are $\rho_{ij} = p_{ij}\,\pi_i$, where π_i is the steady state probability that the head is at cylinder i. The associated decision problem is stated as follows:

PROBLEM ODCA

Given:

M, a positive integer,
ρ_{ij}, $0 \leq i,j \leq M$, such that
$\Sigma_{i,j}\, \rho_{ij} = 1$,
$\Sigma_i\, \rho_{ij} = \Sigma_i\, \rho_{ji}$,
$0 \leq \rho_{ij} \leq 1$,
R, a real number,

is there a permutation function $f: \{0,\ldots,M\} \rightarrow \{0,\ldots,M\}$ such that

$$\sum_{i=0}^{M} \sum_{j=0}^{M} |i-j|\, \rho_{f(i),f(j)} \leq R? \tag{1}$$

THEOREM: ODCA is NP-Complete.

Proof:

I. ODCA $\in$ NP: a nondeterministic algorithm selects f and determines in polynomial time if the inequality (1) holds.

II. Polynomial Transformation:

The *Optimal Linear Arrangement* problem [Garey & Johnson] can be stated as follows:

Given:

$\mathcal{M}$, a positive integer,
e_{ij}, positive integers, $0 \leq i \leq \mathcal{M}$, $j \leq i$
$\mathcal{R}$, a positive integer,

is there a permutation function $f: \{0,\ldots,\mathcal{M}\} \rightarrow \{0,\ldots,\mathcal{M}\}$ such that

$$\sum_{i=0}^{\mathcal{M}} \sum_{j=0}^{i} |i-j| \, e_{f(i),f(j)} \leqslant \mathcal{R}? \tag{2}$$

It will be shown that an instance of the OLA problem can be transformed to an instance of the ODCA problem in polynomial time.

First, the inequality (2) can be rewritten as

$$\frac{1}{2} \sum_{i=0}^{\mathcal{M}} \sum_{j=0}^{\mathcal{M}} |i-j| \, e'_{f(i),f(j)} \leqslant \mathcal{R} \tag{3}$$

where $e'_{ij} = e_{ij}$ if $j \leqslant i$, and $e'_{ij} = e_{ji}$ otherwise. In other words, the matrix $[e'_{ij}]$ is a symmetric matrix formed from the lower triangular matrix $[e_{ij}]$.

Let $\alpha = \sum_{i=0}^{\mathcal{M}} \sum_{j=0}^{i} e_{ij}$. Now, let $\rho_{ij} = e'_{ij}/\alpha$, $M = \mathcal{M}$, and $R = \mathcal{R}$. Since $e'_{ij} = e'_{ji}$, we have $\sum_{i=0}^{M} \rho_{ij} = \sum_{i=0}^{M} \rho_{ji}$. Thus, an arbitrary instance of OLA can be transformed into an instance of ODCA in polynomial time.

■

REFERENCE

[Garey & Johnson] Garey, M. R., and Johnson, D. S., *Computers and Intractability: A Guide to the Theory of NP-Completeness*, W. H. Freeman and Co., San Francisco, 1979.