Supplemental technical report accompanying
Interactive Editing of Lighting and Materials using a Bivariate BRDF Representation (EGSR 2010)

Factoring Lighting, Visibility, and Bivariate Reflectance for Interactive Editing

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Abstract

Exitant radiance depends on lighting, visibility, and surface BRDF. With an appropriate choice of bases, we can write the joint integral of these quantities as an inner product between two terms: one that combines lighting and visibility, and another that depends on the BRDF. This factorization improves interactive editing of the BRDF and viewpoint because the reduced lighting and visibility coefficients do not need to be recomputed unless the lighting is altered.

As described in Section 3.1 of the main paper, we can write the reflection integral as

$$\begin{aligned} I_{n,v,p} &= \int_{\Omega} L_n(\omega) f_v(\omega)(n \cdot \omega) V_p(\omega) d\omega \\ &= \sum_i \sum_j \sum_k L_{n,i} f_{v,j} V_{p,k} C_{ijk}, \end{aligned} \quad (1)$$

where $\{V_{p,k}\}$ are the wavelet coefficients of the visibility, $L_{n,i} = (R_n \tilde{L})_i$, $W_{v,j} = (W_v \tilde{f})_j$, and

$$C_{ijk} = \int_{\Omega} \rho_i(\omega) \rho_j(\omega) \rho_k(\omega) d\omega \quad (2)$$

are the pre-computable tripling coefficients. Our goal is to rewrite the sum in Eq. 1 as

$$I_{n,v,p} = \sum_j (L_n \oplus V_p)_j f_{v,j}, \quad (3)$$

which factors out the BRDF coefficients f_v and collapses the lighting, visibility, and tripling coefficients into *reduced lighting and visibility coefficients* $(L_n \oplus V_p)_j$.

To derive an expression for these coefficients, we begin with the observation of Ng et al. [**?**] that the integral of three 2D Haar wavelet basis functions is non-zero if and only if one of the following conditions hold:

1. All three are scaling functions, in which case $C_{ijk} = 2^{-R}$.
2. All three occupy the same wavelet square and all are different wavelet types, in which case $C_{ijk} = 2^{-R+r}$.
3. Two are identical and the third is either the scaling function or overlaps them at a strictly coarser level, in which case $C_{ijk} = \pm 2^{-R+r}$, where the third function exists at level r.

Given this result, Eq. 1 can be expressed as

$$
\begin{aligned}
I_{n,v,p} = & \sum_{j \in S} 2^{-R} L_{n,j} f_{v,j} V_{p,j} + \\
& \sum_{[s,M] \in D} 2^{-R+r} L_{n,[s,10]} f_{v,[s,01]} V_{p,[s,11]} + \\
& \sum_{[s,M] \in D} 2^{-R+r} L_{n,[s,10]} f_{v,[s,11]} V_{p,[s,01]} + \\
& \sum_{[s,M] \in D} 2^{-R+r} L_{n,[s,01]} f_{v,[s,10]} V_{p,[s,11]} + \\
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& \sum_{[s,M] \in D} 2^{-R+r} L_{n,[s,11]} f_{v,[s,10]} V_{p,[s,01]} + \\
& \sum_{j \in D} L_{n,j} f_{v,j} \mathtt{psum}(V_p, j) + \\
& \sum_{j \in D} L_{n,j} V_{p,j} \mathtt{psum}(f_v, j) + \\
& \sum_{j \in D} V_{p,j} f_{v,j} \mathtt{psum}(L_n, j),
\end{aligned}
$$

where the first summation is over all the scaling coefficients and the remaining summations are over the detail coefficients. Here, `psum` is the *parent sum* operator described by Ng et al. [**?**] which sums the coefficients of all the basis functions that overlap the wavelet square associated with coefficient j.

We can rearrange this sum by factoring out f_v and combining the remaining coefficients and tripling coefficients to obtain

$$
\begin{aligned}
I_{n,v,p} = & \sum_{j \in S} f_{v,j} (2^{-R} L_{n,j} V_{p,j}) + \\
& \sum_{j \in D} f_{v,j} (2^{-R+r} L_{n,j^+} V_{p,j^-}) + \\
& \sum_{j \in D} f_{v,j} (2^{-R+r} L_{n,j} \mathtt{psum}(V_p, j)) + \\
& \sum_{j \in D} f_{v,j} (2^{-R+r} V_{p,j} \mathtt{psum}(L_n, j)) + \\
& \sum_{j \in D} \mathtt{psum}(f_v, j) L_{n,j} V_{p,j}. \qquad (4)
\end{aligned}
$$

If we let $Q(j)$ and $P(j)$ denote the set of basis functions that overlap ρ_j at coarser and finer levels in the wavelet pyramid, respectively, and `sign` correspond to the sign of the coarser basis function j in the region that the finer function j' occupies, then the parent sum operator $\texttt{psum}(f_v, j)$ can be written as

$$\texttt{psum}(f_v, j) = \sum_{j' \in Q(j)} \texttt{sign}(j', j) 2^{(r'-r)} f_{v,j'}.$$

Therefore,

$$\begin{aligned} &\sum_{j \in D} \texttt{psum}(f_v, j) L_{n,j} V_{p,j} \\ &\quad = \sum_{j \in D} \left(\sum_{j' \in Q(j)} (\texttt{sign}(j', j) 2^{(r'-r)}) f_{v,j'} L_{n,j} V_{p,j} \right) \\ &\quad = \sum_{j} f_{v,j} \left(\sum_{j' \in P(j)} (\texttt{sign}(j, j') 2^{(r-r')}) L_{n,j'} V_{p,j'} \right), \end{aligned}$$

where the final summation is over all the wavelet coefficients. This expression may be substituted into Eq. 4 to obtain the desired result:

$$\begin{aligned} I_{n,v,p} = &\sum_{j \in S} f_{v,j} (2^{-R} L_{n,j} V_{p,j}) + \sum_{j \in D} f_{v,j} K_{n,p,j} + \\ &\sum_{j} f_{v,j} \left(\sum_{j' \in P(j)} (\texttt{sign}(j, j') 2^{(r-r')}) L_{n,j'} V_{p,j'} \right). \end{aligned}$$

In the main paper, this same expression is summarized using the following notation:

$$I_{n,v,p} = \sum_{j} (L_n \oplus V_p)_j f_{v,j}, \tag{5}$$

with

$$(L_n \oplus V_p)_j = G_j + \begin{cases} 2^{-R} L_{n,j} V_{p,j} & j \in S \\ K_{n,p,j} & j \in D \end{cases} \tag{6}$$

where

$$G_j = \sum_{j' \in P(j)} (\texttt{sign}(j, j') 2^{(r-r')}) L_{n,j'} V_{p,j'},$$

and

$$\begin{aligned} K_{n,p,j} \quad = \quad & 2^{-R+r} (L_{n,j^+} V_{p,j^-} + L_{n,j^-} V_{p,j^+}) + \\ & 2^{-R+r} (L_{n,j}\, \texttt{psum}(V_p, j) + V_{p,j}\, \texttt{psum}(L_n, j)). \end{aligned}$$