Tradeoffs in Designing Networks with End-to-End Statistical QoS Guarantees

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Abstract— **Recent research on statistical multiplexing has provided many new insights into the achievable multiplexing gain in QoS networks, however, generally only in terms of the gain experienced at a single switch. Evaluating the statistical multiplexing gain in a general network remains a difficult challenge. In this paper we describe two distinct network designs for statistical end-to-end delay guarantees, referred to as *class-level aggregation* and *path-level aggregation*, and compare the achievable statistical multiplexing gain. Each of the designs presents a particular trade-off between the attainable statistical multiplexing gain and the ability to support delay guarantees. The key characteristic of both designs is that they do not require, and instead, intentionally avoid, consideration of the correlation between flows at multiplexing points inside the network. Numerical examples are presented for a comparison of the two designs. The presented class-level aggregation design is shown to yield very high achievable link utilizations while simultaneously achieving desired statistical guarantees on delay.**

Key Words: Statistical Multiplexing, Statistical Service, Quality-of-Service

I. Introduction

In recent years a lot of effort has gone into devising algorithms to support deterministic or statistical QoS guarantees in packet networks. A *deterministic service* [14], which guarantees worst-case end-to-end delay bounds for traffic [7], [8], [27], [28], is known to lead to an inefficient use of network resources [38]. A *statistical* service [14] that makes guarantees of the form

$$Pr[Delay > X] < \varepsilon \,, \tag{1}$$

that is, a service which allows a small fraction of traffic to violate its QoS specifications, can significantly increase the achievable utilization of network resources. Taking advantage of the statistical properties of traffic, a statistical service can exploit *statistical multiplexing gain*, expressed as

$$\left(\begin{array}{l} \text{Resources needed to} \\ \text{support statistical} \\ \text{QoS of } N \text{ flows} \end{array} \right) \ll N \left(\begin{array}{l} \text{Resources needed to} \\ \text{support statistical} \\ \text{QoS of 1 flow} \end{array} \right).$$

Ideally, the statistical multiplexing gain of a statistical service increases with the volume of traffic so that with a high enough level of aggregation the amount of resource allocated per flow is nearly equal to the average resource requirements for a single flow.

Recent research on statistical QoS has attempted to exploit statistical multiplexing gain by taking advantage of knowledge about deterministic bounds on arrivals from individual flows, with limited knowledge about their statistical properties [3], [9], [11], [16], [17], [19], [24], [25], [30], [31], [32]. Under a very general set of traffic assumptions, which are sometimes referred to as 'regulated adversarial traffic', one merely assumes that (1) traffic arrivals from a flow are constrained by a deterministic regulator, e.g., a leaky bucket, and (2) traffic arrivals from different flows are statistically independent. With these general assumptions it has been shown that even if the probability of QoS violations is small, e.g., $\varepsilon = 10^{-9}$, the statistical multiplexing gain at a network node can be substantial [3].

In this paper we are concerned with end-to-end statistical QoS guarantees in a multi-node network under adversarial regulated traffic assumptions. The difficulty of assessing the multiplexing gain in a network environment is that traffic inside the network becomes correlated, and, therefore, the assumption of independence, as made by the regulated adversarial traffic model, no longer holds.

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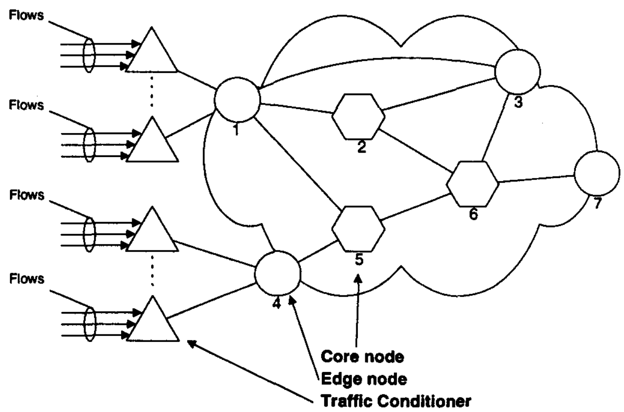


Fig. 1. Network Architecture.

A. Networks with Statistical End-to-End Guarantees

We consider a packet network such as the one shown in Figure 1. The network has two types of nodes, edge nodes and core nodes. Edge nodes are located at the boundary of the network and have links to core nodes or other edge nodes. Core nodes have no links that cross the network boundary. The network distinguishes a fixed number of traffic classes, and flows from the same class have the same characteristics and the same QoS requirements. Traffic which arrives to the network is filtered at a traffic conditioner according to a given traffic profile. Traffic which conforms to the profile is allowed into the network. Traffic which does not conform to the profile is discarded.[1] We assume that nodes execute a scheduling algorithm which can provide rate guarantees to groups of flows [34], [40].

Within this framework, we develop and compare two approaches, referred to as *class-level aggregation* and *path-level aggregation*, for provisioning a network with end-to-end statistical QoS guarantees. Our discussions will investigate the trade-offs presented by the two schemes. A comparison of the approaches will allow us to make recommendations on the design of QoS networks with statistical QoS guarantees.

Overall, we consider statistical QoS guarantees made to traffic on a per-class basis, and not on a per-flow basis. By making QoS guarantees to the aggregate flows from a class and not for specific flows within a class, the design of the core network can be greatly simplified since no per-flow information is required inside the network. The disadvantage of per-class guarantees is that single flows may experience a service which is worse than the service guaranteed to the class as a whole.

B. Related Work

The available literature on statistical QoS is extensive. We refer to [20], [33] for summaries of the state of the art. Here we highlight only a small subset of related literature that focuses on *end-to-end* statistical QoS.

The main difficulty of provisioning statistical QoS for a multi-node network lies in addressing the complex correlation of traffic at downstream multiplexing points. One group of work on end-to-end statistical QoS, attempts to achieve a characterization of correlated traffic inside a network [5], [21], [35], [39]. An alternative approach, which we adopt in Section III, is to reconstruct traffic characteristics inside the network so that arrivals to core nodes satisfy the same properties as the arrivals to an edge node. There are two approaches to reconstruct characteristics of traffic: per-node traffic shaping [15], [41], or per-node delay jitter control [36], [37]. In Section III we take the latter approach.

Another method to achieve statistical end-to-end guarantees is to allocate network capacity for each path or 'pipe' between a source-destination pair in the network, and only exploit the multiplexing gain between the flows on the same path. This method for allocating resource has been considered for use in Virtual Private Networks (VPN) [10] and ATM Virtual Paths [33]. We take such an approach in Section IV.

To our knowledge, there are only a few previous studies which apply the traffic model of adversarial regulated arrivals to multiple node networks. The lossless multiplexer presented in [31] bears similarity to our design for class-level aggregation in Section III, but assumes that routes are such that traffic arrivals at core nodes from different flows are always independent. We relax this assumption in our work, and instead, enforce independence by adding appropriate mechanisms within the network. In [2], probabilistic bounds for end-to-end delay have been derived for networks with coordinated-EDF schedulers, with the extensive examples worked out for the case of on-off traffic sources with deterministic leaky bucket-type bounds on arrivals. Our probabilistic bounds on delay violation do not require EDF-type scheduling.

This paper makes extensive use of results from a recent study [3] which presented a general method to calculate the statistical multiplexing gain at a single node. In particular, we exploit the notion of *effective envelopes* [3], which are functions that provide with high certainty bounds on traffic arrivals. In addition, previous work on rate-based scheduling algorithms with statistical service guarantees in single-node networks is very relevant to our work [15], [29], [42], [43].

The remainder of this paper is structured as follows. In Section II we state our assumptions on traffic arrivals and we introduce the notion of effective envelopes. In Sections III and IV, respectively, we present our two designs for end-to-end statistical QoS and analyze their ability to exploit statistical multiplexing gain. In Section V we evaluate and compare the two designs through a computational study. In Section VI we present conclusions of our work and discuss future research directions. We refer to [22] for an expanded version of this paper.

II. Traffic Arrivals and Effective Envelopes

In this section we present the details of our assumptions for the traffic arrivals. Throughout this paper we will use a fluid-flow interpretation of traffic. We define a function, called an *effective envelope*, which is with high certainty an upper bound on the traffic of multiplexed traffic flows. The concept of effective envelopes will be applied extensively in Sections III and IV. The discussion in this section is based on [3].

A. Regulated Adversarial Traffic

As in all arrival models for a statistical service, the arrivals of a flow are viewed as a random process. Consider a set $\mathcal{C}$ of flows which are partitioned into Q classes, where $\mathcal{C}_q$ denotes the subset of flows from class q. The traffic arrivals from flow j in the interval $[t_1, t_2)$ are denoted by a random variable $A_j(t_1, t_2)$ with the following properties:

(A1) **Additivity.** For any $t_1 < t_2 < t_3$, we have $A_j(t_1, t_2) + A_j(t_2, t_3) = A_j(t_1, t_3)$.

(A2) **Subadditive Bounds.** A_j is bounded by a deterministic subadditive envelope A_j^* as $A_j(t, t+\tau) \leq A_j^*(\tau)$ for all $t \geq 0$ and for all $\tau \geq 0$.[2]

(A3) **Stationarity.** The A_j are *stationary* so that for all $t, t' \geq 0$ we have $Pr[A_j(t, t+\tau) \leq x] = Pr[A_j(t', t'+\tau) \leq x]$.

(A4) **Independence.** The A_i and A_j are stochastically independent for all $i \neq j$.

(A5) **Homogeneity within a Class.** Flows in the same class have identical deterministic envelopes. At each node, flows from the same class have identical delay bounds.

These or similar assumptions are used in many recent works on statistical QoS [3], [9], [11], [16], [17], [19], [24], [25], [30], [31], [32]. The assumptions are very general. Specifically, no assumptions are made on the distribution of flow arrivals, other than that each flow satisfies a worst-case constraint.

Within the constraints of assumptions (A1)–(A5), we consider arrival scenarios where each flow exhibits its worst possi-

[1] As in [13], one may mark out-of-profile traffic with a lower priority, rather than discarding it. However, for the purposes of this study, we do not concern ourselves with out-of-profile traffic.

[2] A function $f : \Re \mapsto \Re$ is subadditive if $f(t_1 + t_2) \leq f(t_1) + f(t_2)$, for all $t_1, t_2 \geq 0$.

ble ('adversarial') behavior. Traffic which obeys the above assumptions is referred to as *regulated adversarial traffic*. Note that even if flows individually behave in a worst-case fashion, as allowed by assumption (A2), the independence assumption (A4) prevents the flows from coordinating (or 'conspiring') to yield a combined or joint worst case behaviour.

B. Effective Envelopes of Aggregate Arrivals

For the calculation of statistical multiplexing gain we will take advantage of the notion of *effective envelopes*, which was recently presented in [3]. Effective envelopes are functions that are, with high probability, upper bounds on multiplexed traffic from a set of flows satisfying the assumptions of adversarial regulated traffic. Effective envelopes have been shown to be a useful tool for calculating the statistical multiplexing gain at a network node.[3]

Consider the set of flows $\mathcal{C}_q$ from a given class q. We use $A_{\mathcal{C}_q}$ to denote the aggregate arrivals from class q, that is, $A_{\mathcal{C}_q}(t, t+\tau) = \sum_{j \in \mathcal{C}_q} A_j(t, t+\tau)$. Also, let N_q denote the number of flows in set $\mathcal{C}_q$. Due to assumption $(A5)$, all flows in the same class have the same subadditive bound. Thus, we use A_q^* to denote the bound of class q with $A_j^*(\tau) = A_q^*(\tau)$ for all $j \in \mathcal{C}_q$.

Definition 1: An effective envelope for $A_{\mathcal{C}_q}(t, t+\tau)$ is a function $\mathcal{G}_{\mathcal{C}_q}$ with:

$$Pr\Big[A_{\mathcal{C}_q}(t, t+\tau) \le \mathcal{G}_{\mathcal{C}_q}(\tau; \varepsilon)\Big] \ge 1 - \varepsilon, \quad \forall\, t,\, \tau \ge 0\,. \quad (2)$$

Due to assumption (A3), an effective envelope provides a bound for the aggregate arrivals $A_{\mathcal{C}_q}$ for all time intervals of length τ, which is violated with probability ε.

Explicit expressions for effective envelopes can be obtained with large deviation results. In this paper, we will use a bound from [3] which is established via the Chernoff Bound. The Chernoff bound for the arrivals $A_{\mathcal{C}_q}$ from $\mathcal{C}_q$ is given by (see [26])

$$Pr[A_{\mathcal{C}_q}(0, \tau) \ge N_q x] \le e^{-N_q x s} M_{\mathcal{C}_q}(s, \tau)\,, \quad (3)$$

where $M_{\mathcal{C}_q}$ is the moment generating function of $A_{\mathcal{C}_q}$ defined as

$$M_{\mathcal{C}_q}(s, \tau) = E[e^{A_{\mathcal{C}_q}(t, t+\tau)s}]\,.$$

In [3], the following bound on the moment generating functions was proven.

Theorem 1: (Boorstyn, Burchard, Liebeherr, Oottamakorn [3]) Given a set of flows $\mathcal{C}_q$ from a single class that satisfies assumptions (A1)–(A5). Then,

$$M_{\mathcal{C}_q}(s, \tau) \quad \le \quad \left[1 + \frac{\rho_q\, \tau}{A_q^*(\tau)}\left(e^{sA_q^*(\tau)} - 1\right)\right]^{N_q}, \quad (4)$$

where $\rho_q := \lim_{\tau \to \infty} A_q^*(\tau)/\tau$.

Using this bound it is possible to show that

$$\mathcal{G}_{\mathcal{C}_q}(\tau; \varepsilon) \ := \ N_q \min(x, A_q^*(\tau))\,, \quad (5)$$

is an effective envelope for $A_{\mathcal{C}_q}$, when x is the smallest number satisfying the inequality

$$\left(\frac{\rho_q \tau}{x}\right)^{\frac{x}{A_q^*(\tau)}} \left(\frac{A_q^*(\tau) - \rho_q \tau}{A_q^*(\tau) - x}\right)^{1 - \frac{x}{A_q^*(\tau)}} \le \varepsilon^{1/N_q}\,. \quad (6)$$

We will use the effective envelope given by Eqs. (5) and (6) in all our numerical examples in Section V.

III. Networks with Class-Level Aggregation ("Jitter Control Method")

In this section we discuss the first of our two approaches to achieve statistical delay guarantees in a multi-node network with regulated adversarial traffic. The key difficulty for analyzing statistical QoS in a network is that, without some kind of intervention, the flows are no longer independent after they have been multiplexed at the edge node. In this section we pursue a solution where each core node has a delay jitter control mechanism that ensures a lower bound on delays [37]. Specifically, if traffic at a node experiences delay which is X seconds shorter than the assigned maximum delay, a delay jitter controller at the next node holds the traffic for X seconds before permitting it to be scheduled. The delay jitter controllers ensure that the traffic arriving at each node has the same statistical properties as the traffic arriving at the network edge. That is, delay jitter controllers restore the statistical independence of arrivals from different flows.

All network nodes run a rate-based scheduling algorithm which guarantees a minimum bandwidth to each traffic class, and each node has a separate buffer of finite size for each traffic class. Traffic which arrives to a full buffer is dropped. The length of the buffer is provisioned such that traffic is dropped only if it violates a given delay bound. Since each network node performs buffering and scheduling on a per-class basis, we refer to this approach as *class-level aggregation*. Figure 2 illustrates how traffic is processed in the network with class-level aggregation, showing the the buffers and jitter controllers for some of the nodes. The conditioners are there to ensure that all traffic flows which arrive to the network satisfy assumption (A2).

We will be able to show that networks with class-level aggregation can guarantee that (1) traffic which is not dropped in the finite-sized buffers meets a given end-to-end delay guarantee, and (2) the drop rate of traffic at each node is bounded.

A. Per Class Delay Jitter Control

As shown in Figure 2, core nodes have a delay jitter control mechanism which ensures that traffic experiences its maximum allocated per-node delay. More precisely, if the route of a flow traverses nodes $m_1, m_2, \ldots, m_n$, with per-node delay bounds $d_{m_1}, d_{m_2}, \ldots, d_{m_n}$, then the delay jitter controller at node m_k $(1 < k \le n)$ holds traffic until the delay of the traffic has a delay equal to $d_{m_1} + d_{m_2} + \ldots + d_{m_{k-1}}$. The implementation of delay jitter control may require time-stamping of packets, and may incur additional buffer requirements [37].

[3]In [3] two notions of effective envelopes are introduced, called local effective envelope and global effective envelope. In this paper, we only use local effective envelopes and refer to them as effective envelopes.

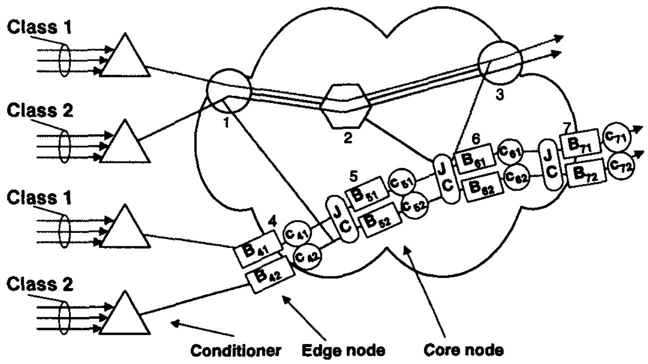


Fig. 2. **Network with class-level aggregation.** At each edge node and each core node, there is one finite-length buffer for each class. Each buffer is served at a fixed rate, and arrivals to a full buffer are dropped. Core nodes have a delay jitter controller, labeled *JC* in the graph, which buffers traffic until it satisfies the maximum allocated per-node delay at the previous node. The figure illustrates the buffers and jitter controllers of nodes 4 – 7. B_{mq} indicates the buffer size and c_{mq} indicates the rate at which the buffer for class q at node m is served.

The jitter control at core nodes ensures that all packets from the same flow experience the same fixed delays (with the exception of delays at the last node). As a consequence, traffic from a flow departing from the delay jitter controller is no worse than the traffic which arrive to the network entrance. Specifically, assuming that there are no losses due to buffer overflows, traffic which satisfies assumptions (A1) – (A5) at the network entrance also satisfies these assumptions at downstream nodes after passing through the corresponding delay jitter controllers. Although losses introduce correlations between flow arrivals, even when jitter control is used, we believe this traffic is bounded by *virtual* arrival processes in the network where no traffic is lost and it satisfies assumptions (A1)–(A5). Since the provisioning is done with respect to these virtual processes, the corresponding delay bounds also apply for the actual traffic with losses. We refer to [22] for more details.

Due to the delay jitter control mechanism, the schedulers are non-workconserving. We believe it is possible to eliminate the delay jitter control, making the schedulers workconserving, and still make the assumptions (A1) – (A5) hold for the traffic inside the network. Instead of holding the traffic at a delay jitter controller for X seconds before sending it to a scheduler, we can add X seconds to the maximum delay of the traffic at the node and immediately send the traffic to the scheduler [2], [14], [23], [36]. As a result, the traffic arrivals are assigned the same maximum delay at a node as if the delay jitter controllers were employed. If the arrivals are scheduled according to their deadlines, then without delay jitter control the traffic will be served in the same order as with delay jitter control. But, without delay jitter control the scheduler can send the traffic earlier whenever the link is idle, possibly resulting in better end-to-end delays. We are developing this approach along the lines of [2] to which we refer for additional information.

B. Rate-Based Scheduling with Per-Class Buffering

As already discussed, we assume that the scheduling algorithm at both edge and core nodes provides per-class queueing and per-class rate guarantees. Class-q traffic which arrives to a scheduler, say at node m, is inserted into a finite buffer with length B_{mq}. Arrivals to a full buffer are dropped and considered lost. The buffer for a class is served at a guaranteed minimal rate, denoted by c_{mq}. Let $\mathcal{C}_{mq}$ denote the set of flows from class q with traffic at node m. We use d_{mq} to denote the delay bound for class-q traffic at node m, $A_{\mathcal{C}_{mq}}$ to denote the aggregate arrivals, and $\mathcal{G}_{\mathcal{C}_{mq}}$ to denote the effective envelope for $\mathcal{C}_{mq}$. Henceforth, we will use $A^*_{\mathcal{C}_{mq}}$ to denote the aggregate worst-case envelope of the traffic in $\mathcal{C}_{mq}$, that is, $A^*_{\mathcal{C}_{mq}}(\tau) = |\mathcal{C}_{mq}| \cdot A^*_q(\tau)$. We select c_{mq} as the smallest number which satisfies

$$\sup_{\tau>0} \left(\mathcal{G}_{\mathcal{C}_{mq}}(\tau;\varepsilon) - c_{mq}\tau\right) \leq c_{mq}d_{mq} , \qquad (7)$$

and we set B_{mq} to

$$B_{mq} = c_{mq}d_{mq} . \qquad (8)$$

The rate c_{mq} in Eqn. (7) is set such that all class-q traffic at node m satisfies delay bound d_{mq}, as long as the arrivals comply to $\mathcal{G}_{\mathcal{C}_{mq}}$, that is, $\mathcal{G}_{\mathcal{C}_{mq}}(\tau) \geq A_{\mathcal{C}_{mq}}(t, t+\tau)$ for all t and τ. Likewise, B_{mq} is set such that traffic is dropped if the delay bound d_{mq} is violated. With these specifications we can state the following properties, proven in [22].

Theorem 2: Given a set of flows $\mathcal{C}_{mq}$ at node m where each A_j with $j \in \mathcal{C}_{mq}$ satisfies assumptions (A1) – (A5), and given a scheduler with per-class buffering and guaranteed service rate for each class, if the c_{mq} and B_{mq} are selected as in Eqs. (7) and (8), respectively, then
1. Traffic which is not dropped meets its delay bound d_{mq}.
2. The rate at which traffic is dropped at node m due to a full buffer is bounded by

$$\varepsilon \cdot \sup_{\tau>0} \left\{ A^*_{\mathcal{C}_{mq}}(\tau) - \mathcal{G}_{mq}(\tau) \right\} , \qquad (9)$$

under the assumption that

$$Pr\left(\sup_{\tau>0} \left\{A_{\mathcal{C}_{mq}}(t-\tau,t) - \mathcal{G}_{\mathcal{C}_{mq}}(\tau)\right\} > 0\right) \approx \sup_{\tau>0} Pr\left(\left\{A_{\mathcal{C}_{mq}}(t-\tau,t) - \mathcal{G}_{\mathcal{C}_{mq}}(\tau)\right\} > 0\right) . \qquad (10)$$

The assumption in Eqn. (10) is similar to an assumption made in [3], as well as in related work [6], [18], [19], [20], [21]. A theoretical justification for this assumption is made in [20], and the assumption has been supported by numerical examples [3], [6], [20].

C. Discussion

There are a number of discussion points to address regarding networks with class-level aggregation as presented in this section.

1. **Loss rate on a path:** Our analysis assumes that the arrivals from a flow j at each node on its path are characterized by $A_{\mathcal{C}_{mq}}$, independent of previous losses. So, our bounds do not quantify the losses that occur at consecutive nodes. As a consequence, we conservatively assume that losses on a path of

nodes occur independent of losses upstream on the path. Consider a sequence of nodes $m_1 \to m_2 \to \ldots \to m_L$, with $\mathcal{C}_{m_l q}$ the set of class-q flows at each node m_l. With the assumptions from Theorem 2, the loss rate for class q on this path is bounded by:

$$\sum_{l=1}^{L} \varepsilon \cdot \sup_{\tau>0} \left\{ A^*_{\mathcal{C}_{m_l q}}(\tau) - \mathcal{G}_{m_l q}(\tau) \right\} . \tag{11}$$

2. **Calculation of c_{mq} and signaling overhead:** The calculation of c_{mq} and B_{mq} is dependent on the cardinality of the set $\mathcal{C}_{mq}$. Each time a new flow is added to the network, the allocation of c_{mq} and B_{mq} must be modified at all nodes on the route of the new flow. However, compared to traditional QoS approaches which maintain per-flow state information, e.g., IntServ [4] and ATM UNI 4.0 [1], the signaling overhead is small.
3. **Dynamic Routing:** In our discussion we have assumed that all traffic of a given class, traveling from specific network ingress to network egress points, traverses the network on the same fixed route. The assumption of fixed routes can be relaxed if mechanisms such as PATH messages in RSVP [12] are used.
4. **Maximum delay bound is incurred at each node:** Due to delay jitter control, traffic experiences worst-case delays at all but the last node on a route, which leads to high buffer requirements. Also, the delay bounds in a network with class-level aggregation are dependent on the number of nodes traversed.
5. **Discrete packet size:** Since actual traffic is sent in discrete-sized packets, performance guarantees given to fluid flow traffic must be matched to guarantees for actual traffic. For rate-based scheduling algorithms the issues involved in transforming guarantees on fluid flow traffic for packet-level traffic are well understood [27], [28], [40]. For example, fluid flow guarantees have been used in the IETF to specify a guaranteed service class for packet-level traffic in the Integrated Services architecture [34].

IV. Networks with Path-Level Aggregation ("Pipe Model")

One possible disadvantage of a network with class-level aggregation, as presented in Section III, is the requirement for delay jitter-control at each node. Aside from being counterintuitive from the perspective of QoS provisioning, delay jitter control leads to large buffer requirements at each node due to the enforcement of maximum delays.

In this section we present an alternative approach, called *path-level aggregation*, which aggregates traffic at a finer level of granularity. Here, flows are multiplexed in the same buffer only if they are in the same traffic class *and* if they traverse the network on an identical end-to-end route. We call an end-to-end route in the network which carries flows from a particular traffic class, a path or 'pipe'.

Figure 3 illustrates a network with path-level aggregation. The figure depicts six paths ("pipes") for two classes. At the network entrance, there is one traffic conditioner for each pipe. The traffic conditioner discards that portion of the aggregate traffic which does not comply to a given policing function. At each network node there is a separate buffer for each pipe with

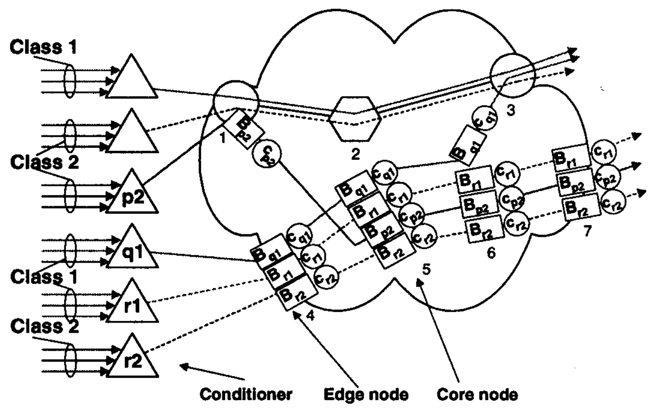


Fig. 3. **Network with path-level aggregation.** An end-to-end path for a traffic class defines a path or 'pipe'. The figure depicts a total of six pipes, and depicts the buffers of four of the pipes, labeled, $p2$, $q1$, $r1$, $r2$. For each pipe, the aggregate traffic is policed at the network entrance by a conditioner. At each node there is a separate buffer for each pipe with traffic at this node. Node buffers are dimensioned such that no overflows occur.

traffic at this node. Thus, flows in the same class are only multiplexed in the same buffer if they have the same end-to-end path. That is, networks with path-level aggregation perform traffic control separately for each 'pipe', and, hence, exploit statistical multiplexing gain only for flows in the same pipe. In contrast to networks with class-level aggregation, network nodes in the path-level scheme do not perform delay jitter control.

A. *Traffic Policing at Traffic Conditioners*

We use $\mathcal{C}_{pq}$ to denote the set of class-q flows which travel on a path p of nodes, where a path is a unique loop-free sequence of nodes which starts and ends with edge nodes.

An important aspect of path-level aggregation is that the aggregate traffic from $\mathcal{C}_{pq}$ which arrives to the network is conditioned using the effective envelope $\mathcal{G}_{\mathcal{C}_{pq}}(.;\tau)$ as policing function. In other words, if $A_{\mathcal{C}_{pq}}(t, t+\tau)$ denotes the aggregate traffic from class $\mathcal{C}_{pq}$ that is admitted into the network in the time interval $[t, t+\tau)$, the policing function $\mathcal{G}_{\mathcal{C}_{pq}}$ ensures that

$$A_{\mathcal{C}_{pq}}(t, t+\tau) \leq \mathcal{G}_{\mathcal{C}_{pq}}(\tau) \qquad \forall t \geq 0, \forall \tau \geq 0 . \tag{12}$$

Traffic in excess of $\mathcal{G}_{\mathcal{C}_{pq}}$ is discarded by the conditioner or at least marked with a lower priority, e.g., as best effort traffic.

For this architecture, Theorem 3 below, proven in [22], provides a bound for the traffic from a pipe which is dropped at the network entrance.

Theorem 3: Given a set of flows $\mathcal{C}_{pq}$, let $A_j(t, t+\tau)$ for $j \in \mathcal{C}_{pq}$ satisfy assumptions (A1) – (A5). If the arrivals $A_{\mathcal{C}_{pq}}$ are policed according to Eqn. (12), then the rate of dropped traffic is bounded by

$$\varepsilon \cdot \sup_{\tau>0} \left\{ A^*_{\mathcal{C}_{pq}}(\tau) - \mathcal{G}_{\mathcal{C}_{pq}}(\tau) \right\} , \tag{13}$$

with the assumption that

$$Pr\left(\sup_{\tau>0}\{A_{\mathcal{C}_{pq}}(t-\tau,t)-\mathcal{G}_{\mathcal{C}_{pq}}(\tau)\}>0\right)\approx \sup_{\tau>0} Pr\left(\{A_{\mathcal{C}_{pq}}(t-\tau,t)-\mathcal{G}_{\mathcal{C}_{pq}}(\tau)\}>0\right) . \quad (14)$$

B. Scheduling at Edge Nodes and at Core Nodes

With path-level aggregation, allocation of bandwidth and buffer space at a node is done separately for each 'pipe'. At each node on the route of a pipe, the same buffer size and bandwidth is allocated. We use c_{pq} to denote the rate which is allocated at each node on the route of the pipe, and we use B_{pq} to denote the reserved buffer space. We set c_{pq} as the smallest number which satisfies

$$\sup_{\tau>0}\left(\mathcal{G}_{\mathcal{C}_{pq}}(\tau;\varepsilon)-c_{pq}\tau\right)\le c_{pq}d_{pq} , \quad (15)$$

where d_{pq} is the end-to-end delay bound for $\mathcal{C}_{pq}$, and we set the buffer space B_{pq} according to

$$B_{pq}=c_{pq}d_{pq} . \quad (16)$$

The next theorem, proven in [22], states properties for the end-to-end performance of flows in $\mathcal{C}_{pq}$ with end-to-end delay bound d_{pq}.

Theorem 4: Given a set of flows $\mathcal{C}_{pq}$ where each A_j with $j\in\mathcal{C}_{pq}$ satisfies assumptions (A1) – (A5), and assuming the existence of policing functions at network ingress points that enforce Eqn. (12), if c_{pq} and B_{pq} are allocated as given in Eqs. (15) and (16) at each node on the route taken by flows in $\mathcal{C}_{pq}$, then
1. No traffic is dropped inside the network.
2. The end-to-end delay of traffic satisfies delay bound d_{pq}.

Class-level aggregation and path-level aggregation instantiate a fundamentally different trade-off between the ability to provision low delay bounds and the ability to yield a high multiplexing gain. Since class-level aggregation multiplexes all flows which are in the same class, whereas path-level aggregation multiplexes only flows in the same 'pipe', we expect the multiplexing gain of class-level aggregation multiplexes to be better than that of path-level aggregation. On the other hand, class-level aggregation requires a jitter control mechanism at each node. The delay jitter control ensures that traffic experiences the maximum node delay at each node (except the last node on the path).

V. Numerical Evaluation

In this section, we present numerical examples to compare the ability of class-level and path-level aggregation to support statistical end-to-end delay guarantees. Our discussion so far has pointed out the trade-offs presented by the two approaches. Class-level aggregation multiplexes larger groups of flows than the path-level approach and is thus expected to yield a better multiplexing gain. On the other hand, delay jitter control in networks with class-level aggregation may result in higher delay bounds.

In our numerical examples, we perform a comparison of four different approaches for provisioning QoS.

TABLE I
Parameters of four traffic classes.

Class	Type		Burst Size σ_q (bits)	Mean Rate ρ_q (Mbps)	Peak Rate P_q (Mbps)	End to End Delay d_q (msec)
	burst-iness	delay				
1	low	low	10^4	0.15	6.0	10
2	low	high	10^4	0.15	6.0	40
3	high	low	10^5	0.15	6.0	10
4	high	high	10^5	0.15	6.0	40

- **Deterministic QoS:** Here, all nodes implement a rate-based scheduling algorithm, such as GPS [27]. If $\mathcal{C}_{mq}$ is the set with class q flows at node m, then the rate allocated for this class at node n is the smallest value c_{mq} such that $\sup_{\tau>0}\left\{A^*_{\mathcal{C}_{mq}}(\tau)-c_{mq}\tau\right\}\le c_{mq}d_q$, where $A^*_{\mathcal{C}_{mq}}(\tau)=|\mathcal{C}_{mq}|\cdot A^*_q(\tau)$. From [28] we know that, if all nodes m allocate a bandwidth of c_{mq}, all class-q traffic will satisfy an end-to-end delay bound of d_q.
- **Statistical QoS with class-level aggregation:** The bandwidth and buffer allocation is as given in Eqs. (7) and (8). The QoS guarantee for class-q is as stated in Theorem 2. The calculation of the effective envelope is done as discussed in Section II-B.
- **Statistical QoS with path-level aggregation:** The bandwidth and buffer allocation is as given in Eqs. (15) and (16). The QoS guarantee for class-q is as stated in Theorems 3 and 4.
- **Average Rate Allocation:** This scheme allocates bandwidth equal to the average traffic rate for flows. So, if $\mathcal{C}_{mq}$ is the set with class q flows at node m, node m allocates a rate equal to $|\mathcal{C}_{mq}|\rho_q$ for class q. (Recall that $\rho_q=\lim_{\tau\to\infty}A^*_q(\tau)/\tau$). Average rate allocation only guarantees finite delays and average throughput, but no per-flow or per-class QoS.

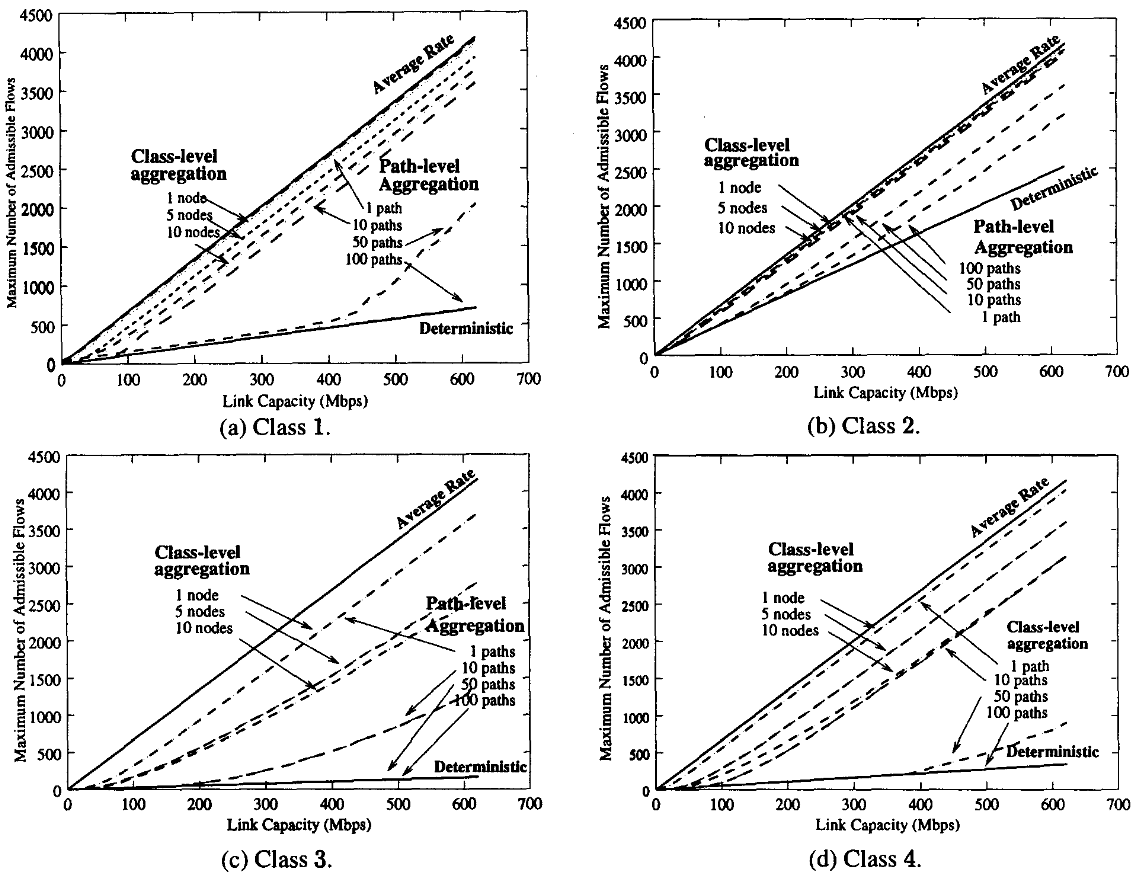
As an admission control condition, we require for all schemes above that the total allocated bandwidth on a link must not exceed the link capacity.

We consider four classes of traffic, and assume that traffic flows in each class are regulated by a peak-rate constrained leaky bucket with parameters (σ_q,ρ_q,P_q), and deterministic envelope $A^*_q(\tau)=\min(P_q\tau,\sigma_q+\rho_q\tau)$ for class q. The parameters for the flow classes are given in Table I. We set $\varepsilon=10^{-6}$ in all our examples. The parameters are similar to those used in other studies on regulated adversarial traffic [11], [24], [30].

A. Maximum Number of Admissible Flows

In Figures 4(a)-(d) we plot the maximum number of flows which can be provisioned with QoS on a link in the network, as a function of the link capacity. We vary the link capacity in the range 1 Mbps – 622 Mbps. The figures show the maximum number of flows which can be admitted on a link. The average rate allocation serves as an upper bound and the peak rate allocation as the lower bound for the number of flows on a link.

For class-level aggregation, first recall from our discussions in Section III-C and Section IV-B that, due to delay-jitter control, the end-to-end delay bound is dependent on the number of links. So, if the end-to-end delay bound is given by



(a) Class 1. (b) Class 2. (c) Class 3. (d) Class 4.

Fig. 4. Maximum number of admissible flows.

$d_q = 10$ msec, and the path length is given by L, the per-node delay bound is given by $10/L$ msec (assuming that the delay budget is evenly divided among all nodes). Thus, the longer the route of a flow, the smaller the per-node delay bound, and, consequently, the smaller the total number of flows that can be accommodated on a link. In Figures 4(a)-(d), we consider path lengths equal to $L = 1, 5$, and 10 nodes. As the figures show, even for longer path lengths, class-level aggregation yields a significant multiplexing gain. For traffic classes 1 and 2, which exhibit lower burstiness, the number of admissible flows with class-level aggregation is close to that of the admissible number of flows with an average rate allocation, even when the length of the route grows as large as 10 nodes.

For path-level aggregation, the achievable multiplexing gain is dependent on the number of paths (pipes) that extend from a given ingress point. In our examples, we only consider one class at a time, so the number of paths at a node is given by the number of different end-to-end routes. In a network with M edge nodes, the maximum number of paths at any core node is given by $O(M^2)$ Since path-level aggregation only performs multiplexing of flows on the same path, the number of flows which can be multiplexed on a link decreases with M. In Figures 4(a)-(d), we show the results for path-level aggregation with $1, 10, 50$, and 100 paths. (Note that the maximum number of flows that can be supported with path-level aggregation with only 1 path is identical to class-level aggregation with a 1 hop route.) Figures 4(a)-(d) show that the maximum number of flows which can be provisioned with QoS quickly deteriorates as the number of paths increases. For 100 paths in the network, we observe for all traffic classes that path-level aggregation accommodates the same number of flows as deterministic QoS.

In summary, the performance of path-level aggregation quickly decreases as the number of paths increases. Since the number of paths grows (in the worst case) with the square of the number of edge nodes in a network, path-level aggregation appears to be a viable technique only in small networks. Class-level aggregation, on the other hand, even though it is sensitive to the length of the routes, yields a very high statistical multiplexing gain.

B. Comparison of the Traffic Loss Rate

We now compare the expected loss rate of a flow with the chosen set of parameters. Recall that the loss rate as given in Eqs. (11) and (13) is for traffic classes, and not for single flows. Making QoS guarantees to aggregate flows yields a simple network core, since no per-flow information is required. On

TABLE II

NORMALIZED LOSS RATE PER FLOW. (Note that the loss rate for class-level aggregation is dependent on the path length.)

	Class-level Aggregation			Path-level Aggregation
	1 node	2 nodes	10 nodes	
Classes 1, 2	$6.7 \cdot 10^{-8}$	$1.3 \cdot 10^{-7}$	$6.7 \cdot 10^{-7}$	$6.7 \cdot 10^{-8}$
Classes 3, 4	$6.7 \cdot 10^{-7}$	$1.3 \cdot 10^{-6}$	$6.7 \cdot 10^{-6}$	$6.7 \cdot 10^{-8}$

the other hand, single flows may experience a service which is worse than the service guaranteed to the class as a whole. To obtain a measure on the loss rate, we normalize the loss rate of a class by the long time average of the expected traffic in the class. (We point out that this 'normalization' does not give a precise loss rate.) Under the assumption that $\mathcal{C}_{mq}$ is the set of class-q flows at all nodes, we obtain with Eqn. (11) a (normalized) loss rate of

$$\text{Loss Rate}^{class} < \frac{1}{\rho_q \cdot |\mathcal{C}_{mq}|} \cdot L \cdot \varepsilon \cdot \sup_{\tau>0} \left\{ A^*_{\mathcal{C}_{mq}}(\tau) - \mathcal{G}_{mq}(\tau) \right\} . \quad (17)$$

In our example, since $A^*_{\mathcal{C}_{mq}}(\tau) = |\mathcal{C}_{mq}| \cdot \min(P_q\tau, \sigma_q + \rho_q\tau)$ and with $\mathcal{G}_{mq}(\tau) \geq \rho_q\tau \cdot |\mathcal{C}_{mq}|$ we obtain the bound

$$\text{Loss Rate}^{class} < \frac{\sigma_q}{\rho_q}\varepsilon L . \quad (18)$$

The same consideration for the path-level scheme yield

$$\text{Loss Rate}^{path} < \frac{\sigma_q}{\rho_q}\varepsilon . \quad (19)$$

In Table II we give the results for bounds on the normalized loss rate for all classes used in our examples. The total loss rate is small in all cases, and is of the same order as ε.

C. Sensitivity of Path-Level Aggregation to the Number of Paths

In Figure 4 we saw that path-level aggregation resulted in relatively poor achievable utilization at a link, when the number of paths (routes) in the network was high. Here we provide more insight into the sensitivity of path-level aggregation towards an increase of the number of paths.

We use as performance measure the rate that is allocated per flow to support the desired QoS level on a saturated link. We call this measure the *effective rate* of a flow. The effective rate is determined by first calculating the maximum number of flows which can be provisioned on a link with a desired QoS level (deterministic, statistical with class-level aggregation, statistical with path-level aggregation), and then dividing the link capacity by the number of flows.

Figures 5(a)-(d) show the results for traffic classes 1 through 4. For illustrative purposes, we plot the values of 1/(effective rate) as a function of the link capacity and as a function of the number of paths.

Figures 5(a)-(d) show the results for deterministic QoS and for the two statistical QoS schemes considered here. A larger value of 1/(effective rate) indicates a better statistical multiplexing gain. The effective rate of a flow with deterministic QoS is not sensitive to increases in link capacity or in the number of paths. For QoS with class-level aggregation, the statistical multiplexing gain increases with the link capacity, but does not increase with the number of paths. However, as discussed earlier, the achievable statistical multiplexing gain is dependent on the length of a route. In Figures 5(a)-(d) we include plots for route lengths of 1 node, 2 nodes, and 10 nodes.

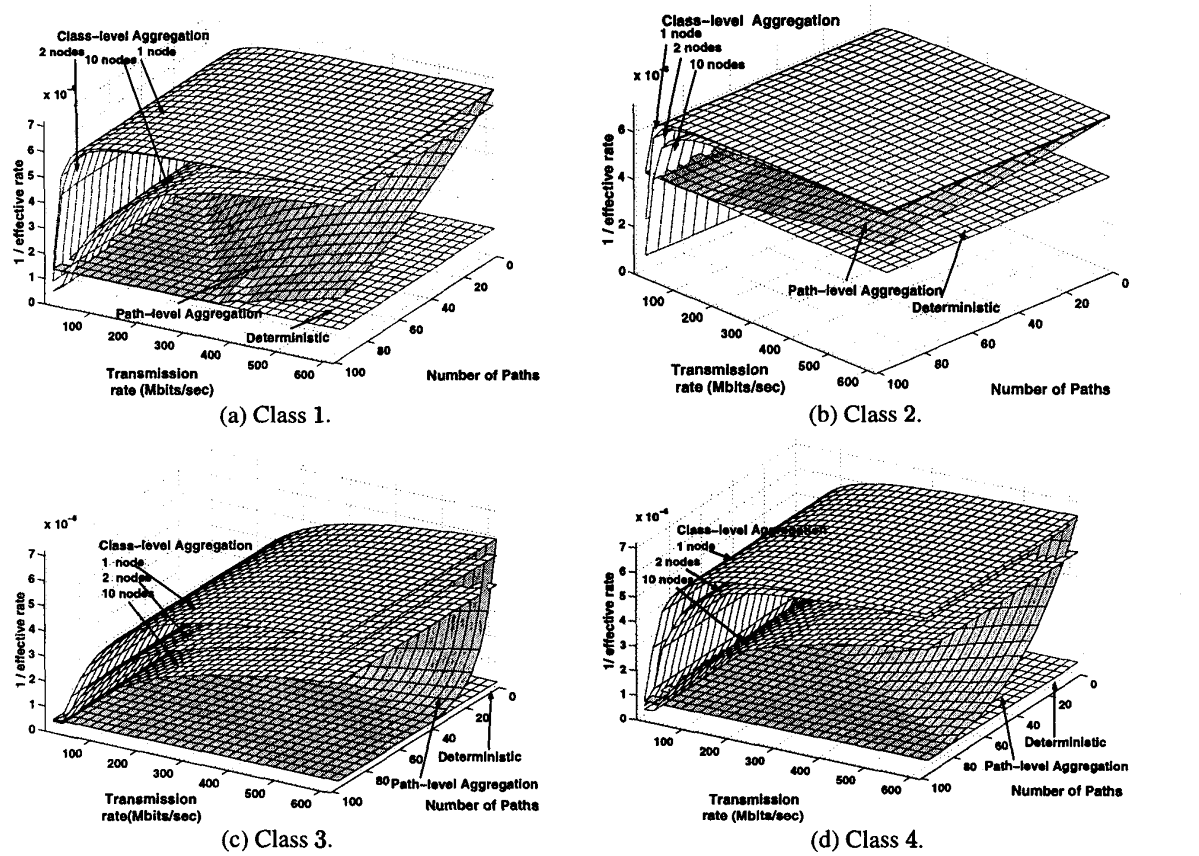
The results for path-level aggregation are perhaps the most interesting aspect of Figures 5(a)-(d). We see that a high level of statistical multiplexing gain is achievable only if the link capacity is high, and the number of paths is small. Since the number of paths can grow as fast as the square of the number of edge nodes, the multiplexing gain achievable in network deteriorates quickly as the number of paths grows large.

VI. Discussion and Conclusions

In this paper we have studied two designs for networks with end-to-end statistical service guarantees: class-level aggregation (Section III) and path-level aggregation (Section IV). The class-level approach can achieve a very high level of aggregation, resulting in a better statistical multiplexing gain; however, this comes at the expense of requiring delay jitter control for restoring the statistical independence of the flows at each node. Thus, it is required in this scheme to assign a maximum allowable delay to each node on the path for an end-to-end flow. The need for jitter control is admittedly a counterintuitive notion which we believe is justified by the high level of achievable statistical multiplexing gain. In the alternative, path-level scheme, there is no need for delay jitter control, since flows in this design are multiplexed only if they are of the same class *and* they share the same path through the network. Consequently, statistical multiplexing gain is perceived only at the network ingress points, at a much lower level of aggregation. The tradeoff between the two designs is one of enforced delay (and design complexity) in the form of delay jitter control for high levels of achievable statistical multiplexing gain.

Our numerical results indicate that the increased statistical multiplexing gain achievable with class-level aggregation is worth the price paid in terms of enforced delay. In the path-level aggregation design, as the number of paths in the network increases, the achievable statistical multiplexing gain quickly diminishes to the achievable multiplexing gain in making deterministic (worst-case) QoS guarantees. Thus, we assert that the class-level scheme is the preferred approach for implementing statistical end-to-end delay guarantees.

There are a number of important issues that have to be addressed before the class-level design can be implemented in a real network. First, we must reconcile our assumption of fixed routes with dynamic routing which is prevalent in the Internet today. We need to develop appropriate data structures and algorithms that allow rapid computation of guaranteed rates and buffer allocations within the network. We need to formulate a packet-level version of our fluid-model constructs, in order for a real implementation to be possible. Here, we expect that the well-known approach from [28] will be sufficient. Finally, we plan to address the issue of how to characterize traffic flows in terms of worst-case bounding functions. Our development so far has rested on the assumption that a worst-case, subadditive bounding function is available for each traffic flow. If bounding functions for flows are not available a priori, it be-



(a) Class 1. (b) Class 2.

(c) Class 3. (d) Class 4.

Fig. 5. Results for $1/c^{eff}$, where c^{eff} is the effective rate of a flow as a function of the link capacity and the number of paths in the network.

comes essential to estimate these bounds from data. Estimating a traffic bound from measurements is one topic of our ongoing research.

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