# A QUEUEING MODEL FOR TOKEN RINGS WITH PRIORITIES

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Computer Science Report No. TR-87-10
March 1987

Submitted to *IEEE Transactions on Communications*, March 1987.

# A Queueing Model for Token Rings with Priorities

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*Abstract* — We present an analytic queueing model which predicts delays for priority traffic on token rings which use a reservation priority scheme (such as IEEE 802.5 and SAE AE-9B). Our model consists of five components: queueing delay, priority wait time, priority block time, transmission delay, and ring latency delay. We provide formulae and derivations for each. The model has been verified by comparison with a simulation of the IEEE 802.5 token ring. Although we model single-packet-per-token service, we show this to be operationally justified.

## I. Priority Queueing Model

### A. *Introduction*

Many token ring network protocols provide the user with multiple message priorities. There are two main mechanisms for priority schemes: (1) some form of service time limitation to control the amount of service allocated to the various priorities, and (2) a priority reservation scheme where the token indicates the global ring priority level. In this paper we develop a queueing model for a priority scheme using a reservation mechanism, and validate our model by comparison to simulation results.

### B. *Definitions*

We introduce the following definitions:

$N \equiv$ number of active ring stations

$n \equiv$ number of active priorities at each station, $n \geq 1$

$\mu \equiv$ average service rate for packets of all priorities

$\Lambda \equiv$ a station's total packet arrival rate

$\lambda_i \equiv$ a station's arrival rate for priority $i$ packets, $\sum_{i=1}^{n} \lambda_i = \Lambda$

$\Phi \equiv$ a station's network offered load of all priorities, $\Phi = \frac{\Lambda}{\mu}$

$\rho_i \equiv$ a station's offered load of priority $i$, $\rho_i = \frac{\lambda_i}{\mu}$, $\Phi = \sum_{i=1}^{n} \rho_i$

$\sigma_i \equiv$ the load affecting the priority wait time of a packet of priority $i$, $\sigma_i = \sum_{k=i}^{n} \rho_k$

$\gamma \equiv$ total ring latency (propagation time plus latency buffers)

$E[C] \equiv$ expected token cycle time $= \frac{\gamma}{1 - N\Phi}$, $N\Phi < 1$

$E[Q] \equiv$ expected initial queueing delay for packets of all priorities

$E[W_i] \equiv$ expected priority wait time for a packet of priority $i$

$E[\Delta_i] \equiv$ expected delay incurred by a priority $i$ packet when it is blocked from transmission

$P(X = x) \equiv$ the probability of $X$ distinct reservations out of $x$ total reservations in one token cycle

Equation (1) is the total expected delay $E[D_i]$ experienced by a packet of priority $i$:

$$E[D_i] = E[Q] + E[W_i] + E[\Delta_i] + \frac{1}{\mu} + \frac{\gamma}{2} \qquad (1)$$

From the above definitions, $E[D_i]$ is the sum of: the queueing delay, $E[Q]$, the priority wait time for priority $i$, $E[W_i]$, the delay attributable to the packet being blocked, $E[\Delta_i]$, and the time needed to transmit and propagate the packet, $\frac{1}{\mu} + \frac{\gamma}{2}$. Each of the above terms is derived in the sections which follow.

There are two key concepts to the model: *priority wait time* and *priority block time*. *Priority wait time* is a special case of network access delay, namely, the time a packet must spend waiting for a usable token once it has reached the head of its (conceptual) priority queue. This time is separate from the queueing delay of a packet, which is uninfluenced by its priority. *Priority block time* is the added delay experienced by a packet when it is blocked from transmission by the presence of a packet of higher priority in that station's transmission queue.

*C. Queueing Delay, E [Q ]*

It is shown in [3] that the delay experienced by a packet while waiting for network access can be divided into two parts: the time spent in the queue and the time spent waiting for the token. We utilize this decomposition principle to separate the initial queueing delay from the priority wait time.

Queueing delay is dependent upon the station's total offered load ($\Phi$) and, since this is the initial unblocked queueing delay, is common to packets of all priorities. From the Pollaczek-Khinchine formula for the expected delay of a packet in an M/G/1 queue, the queueing delay is given by:

$$E[Q] = \frac{\Lambda \frac{1}{\mu^2}}{2(1-\Phi)}, \quad N\Phi < 1 \tag{2}$$

*D. Priority Wait Time, E [$W_i$]*

Priority wait time is the delay experienced by a packet while waiting for a usable token, and is incurred only when a packet has reached the head of its priority queue. Using results from [1], [2], and [4], the priority wait time of a single packet of priority $i$ is:

$$E[W_i] = E[C](N-1)\Lambda(E[C] + \frac{1}{\mu})P(X=x) + \frac{\sigma_i (N-1)\frac{1}{\mu} + \gamma(1-\sigma_i)^2}{2(1-N\sigma_i)(1-\sigma_i)}, \quad \Lambda E[C] < 1 \tag{3}$$

The first term, $E[C](N-1)\Lambda(E[C] + \frac{1}{\mu})P(X=x)$, represents the time needed for the reservation mechanism to operate. The number of reservations made in one token cycle time is dependent on the total offered load contribution of the non-transmitting stations, hence the term $(N-1)\Lambda$. These reservations are made in one token cycle plus the time required to transmit the current packet, which is $E[C] + \frac{1}{\mu}$. One token cycle is required (on average) to recognize a priority reservation. Therefore, it is necessary to multiply the number of *distinct* attempted reservations, $(N-1)\Lambda(E[C] + \frac{1}{\mu})P(X=x)$, by one token cycle time in order to account for the delay caused by the reservation mechanism where $(N-1)\Lambda(E[C] + \frac{1}{\mu})$ is the *total* number of reservations attempted. However, since more than one reservation for the same priority has the identical effect as one, $P(X = x)$ gives the probability that all attempted reservations are for different priority levels, and is an ordinary sampling with replacement computation.

However, this station's request may not be the highest priority request recorded on this token cycle. Thus the second term, which is a function of priority level $i$, represents the expected wait to receive a usable token after a reservation has been made, i.e., a token of priority less than or equal to $i$. The second term is a restatement of the work in [1], modified to account for priorities.

*E. Priority Block Time, $E[\Delta_i]$*

The probability that a packet is blocked from transmission is of crucial importance. From Little's Law, the number of packet arrivals of priority $i$ at one station during one token cycle is $\lambda_i E[C]$. If $\lambda_i E[C] < 1$, it is also the probability of there being a priority $i$ packet present upon token arrival. If we further restrict the total arrival rate to be $\Lambda E[C] < 1$, then $\Lambda E[C]$ is the probability of there being any packet at the station upon token arrival. It follows that $E[C]\sum_{i=1}^{n}\lambda_i = \Lambda E[C] < 1$. In the following discussion, we explicitly assume that $\Lambda E[C] < 1$, and this assumption is justified in section H.

The probability that a packet of priority $i$ is blocked from transmission at token reception due to the presence of a packet of higher priority is equal to the probability that a packet of higher priority is present, which is:

$$P_i = E[C] \sum_{k=i+1}^{n} \lambda_k, \quad \Lambda E[C] < 1 \tag{4}$$

Equation (4) is not sufficient, however, since it gives the probability that a packet is blocked at only a single token reception. Since a packet has a probability $P_i$ of being blocked on one token reception, it has probability $P_i^2$ of being blocked on two token receptions, $P_i^3$ of being blocked on three, etc., leading to

$$P_i + P_i^2 + P_i^3 + \cdots = \frac{P_i}{1 - P_i} \tag{5}$$

which is the expected number of token cycles for which a packet of priority $i$ will be blocked.

A packet experiences added delay if it is blocked. We compute the mean delay due to blocking by multiplying the expected number of token cycles for which the packet is blocked by the delay incurred when a packet is blocked on one token cycle. This is

$$E[\Delta_i] = \frac{P_i}{1 - P_i} \sum_{k=i}^{n} (E[C] + \frac{1}{\mu} + E[W_k] + E[Q]) \qquad (6)$$

In equation (6), a blocked packet must wait $E[C]$ for the token to complete another token cycle before this packet can again attempt access to the network. It must also wait for the blocking packet to be transmitted, which is $\frac{1}{\mu}$. Since the packet must now bid for the token again, another priority wait time $E[W_k]$ must also be added (i.e., the reservation mechanism must begin anew). Finally, another queueing delay $E[Q]$ is added since the effect of a packet being blocked is identical to its preemption in a priority queue. The summation in equation (6) is required because if a packet is blocked, the blocking packet might also be blocked. It is therefore necessary to incorporate all the delays incurred by higher priority blocked packets. Note that $E[W_k]$ is dependent on the variable of summation, whereas $P_i$ is not. This is because we are computing the probability that a packet of priority $i$ is blocked, but priority wait times vary according to the priority of the packet being blocked. Also note that as $P_i \to 1, E[\Delta_i] \to \infty$, which is the desired result for lower priority packets when faced with increasing offered load.

*F. Transmission and Latency Delay*

When a packet finally gains access to the network, it is necessary to add the delay of transmission, $\frac{1}{\mu}$. Assuming that all packet destinations are equally likely, the latency to reach the destination will, on average, be half the ring latency, or $\frac{\gamma}{2}$.

*G. Final Result*

Adding the queueing delay, priority wait time, and priority block time to the transmission and latency delay yields equation (1). Using terms we have defined in section I-B, equation (1) can be expanded to a form more suitable for programming (we have an implementation in C of less than 75 lines):

$$E[D_i] = \frac{\sigma_i (N-1)\frac{1}{\mu} + \gamma(1-\sigma_i)^2}{2(1-N\sigma_i)(1-\sigma_i)} + \frac{\gamma}{1-N\Phi}(N-1)\Lambda\left[\frac{\gamma}{1-N\Phi} + \frac{1}{\mu}\right] P(X=x) +$$

$$\frac{\Lambda\frac{1}{\mu^2}}{2(1-\Phi)} + \frac{1}{\mu} + \frac{\gamma}{2} + \frac{\frac{\gamma}{1-N\Phi}\sum_{k=i+1}^{n}\lambda_k}{1-\frac{\gamma}{1-N\Phi}\sum_{k=i+1}^{n}\lambda_k} \times \sum_{k=i}^{n}\left[\frac{\sigma_k(N-1)\frac{1}{\mu} + \gamma(1-\sigma_k)^2}{2(1-N\sigma_k)(1-\sigma_k)} + \right.$$

$$\left.\frac{\gamma}{1-N\Phi}(N-1)\Lambda\left[\frac{\gamma}{1-N\Phi} + \frac{1}{\mu}\right] P(X=x) + \frac{\gamma}{1-N\Phi} + \frac{\Lambda\frac{1}{\mu^2}}{2(1-\Phi)} + \frac{1}{\mu}\right] \quad (7)$$

*H. Single-packet-per-token Service*

The above discussion was predicated on a station being able to emit either zero or one packet upon token receipt, which is the defined mode of operation in the SAE AE-9B High Speed Ring Bus protocol [5]. However, the IEEE 802.5 token ring [6] allows multiple packets to be transmitted per token receipt, provided other restrictions on token rotation time and packet length are met, although currently IBM's implementation of 802.5 only allows single-packet-per-token service.

Even for the IEEE token ring, the assumption of single-packet-per-token (SPPT) service is easily justified. In a network of $N$ stations, if $\Lambda$ is the total arrival rate of all packets at a station, $\Phi$ is the total offered load contributed by a station, and $\gamma$ is the no-load latency of the ring (also called the empty ring walk time), then from Little's Law, the expected length $E[L]$ of a station's queue upon token arrival is

$$E[L] = \frac{\Lambda\gamma(1-\Phi)}{1-N\Phi}, \quad N\Phi \le 1 \quad (8)$$

The factors which would tend to make the queue grow are: large offered load, a small number of stations for a given offered load, short packets, and slow ring transmission speed. Assuming $N\Phi = 0.95$, $N = 3$, $\gamma = 30$ μsec, ring transmission speed is 1 Mbps, and frames are 32 octets in length, $E[L] \approx 0.5$. In this example, the ring size of $N = 3$ is artificially small, and the frame size of 32 octets is considerable less than the maximum frame size of SAE-9B or of IEEE 802.5. Also, the 1 Mbps ring transmission speed is slow when compared to the 4 Mbps capacity of the higher performance version of IEEE 802.5, or to the 100 Mbps capacity of SAE-9B. Even for this extreme example, $\Lambda E[C] < 0.75$, which suggests that our model's restriction is quite realistic. Thus for "normal" configurations a token will arrive to find an expected queue length of significantly less than one, justifying our SPPT assumption.

### *I. Multiple-packet-per-token Service*

Derivation of a multiple-packet-per-token model (MPPT) is simple in theory but complex in practice, and is a generalization of the model presented here. The calculation of $P_i$ changes, since it becomes a combinatorial problem of expressing the number of ways in which a packet can be blocked, along with the associated blocking probabilities. The $E[W_i]$ term must also be modified since a packet may be delayed for less than one token cycle time (i.e., only a few packet transmission times). It is clear, however, that a MPPT service discipline will reduce delays and diminish the impact of the priority scheme, and this is in accordance with simulation results. Therefore, the "interesting" cases are those which follow a SPPT service discipline.

## II. Numerical Results

### *A. General Performance*

Simulation results for a 1 Mbps IEEE 802.5 token ring in [4] show that for any particular offered load, achieved throughput for the token ring decreases as packet sizes become shorter, and that this loss of throughput is exaggerated when the number of ring stations is small. It was observed that the effect of MPPT service only had observable effect when the number of ring stations was less than 40; the maximum difference in delay recorded was a factor of two for a simulation of 3 stations. It was also shown that for a fixed offered load, delay is inversely proportional to packet size (i.e., small packets are less efficient than large packets). In this section we deal only with the "interesting" case where the number of stations is small, the packet length is short, and service is restricted to SPPT.

*B. Comparison of the Queueing Model with Simulation Results*

We simulated an IEEE 802.5 token ring with a transmission rate of 1 Mbps. The configuration used ring repeater latencies of 1 bit time per station and packet sizes of 32 octets. Offered loads were varied from 5% to 95% of capacity, and arrivals were Poisson. In the graphs shown, the load is equally distributed among all eight priorities.

Figure 1 compares the queueing model prediction with simulation results for a 40 station network operating with only a single priority; the differences are negligible. Figures 2, 3, and 4 compare the priority queueing model prediction with simulation results when using SPPT service for 3, 10 and 20 stations respectively. The errors seen at high loads are due to the fact that the model is not accounting for the effective stabilization of offered load reflected in the increasing delays of low priority packets. When values outside of this error range are considered (generally up through 90% offered load), agreement between the queueing model and simulation are within one packet transmission time (256 μs) for all priority classes (within one-half of one packet transmission time in over 80% of the cases studied).

*C. Limitations*

The applicability of this model is limited by two restrictions. First, the total offered load at a station is limited by $\Lambda E[C] < 1$ so that SPPT service is justified. Second, we have not accounted for the special case when packets are short relative to ring latency; a station may delay until the packet header returns so that the new token will be issued at the highest reserved priority. We do not believe that either of these restrictions meaningfully inhibit the usefulness of the model.

## III. Conclusions

We conclude that the queueing model presented here is a good predictor of delay for priority traffic on token rings which utilize a priority reservation scheme. The model has been verified against a simulation of the IEEE 802.5 token ring and agreement is generally within one packet transmission time. Although our model is restricted to SPPT service and $\Lambda E[C] < 1$, we have shown that both assumptions are operationally justified.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support of the Institute of Information Technology of the Virginia Center for Innovative Technology.

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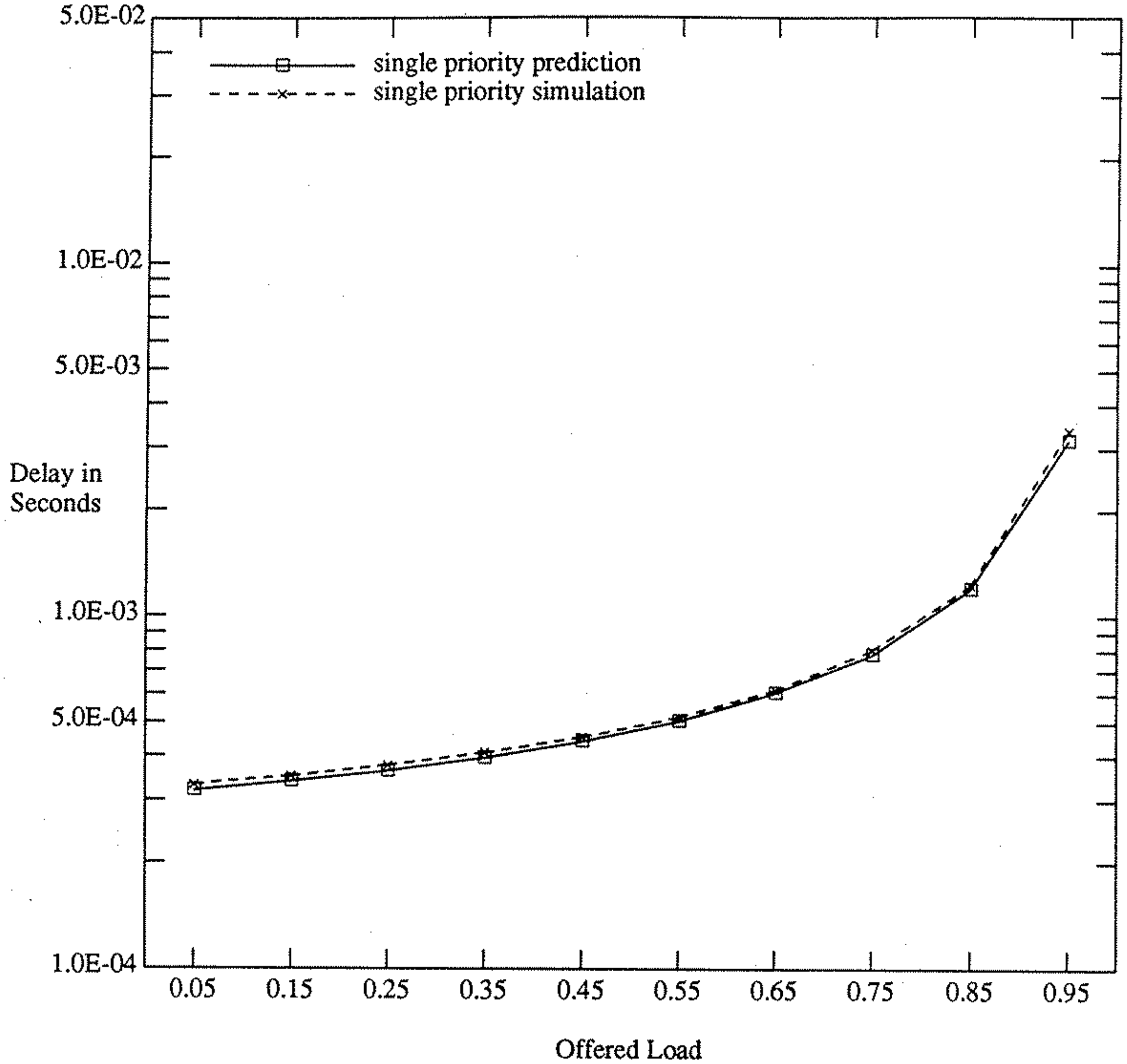


**Figure 1**
**Single Priority Queueing Model vs. Simulation for 40 Stations**

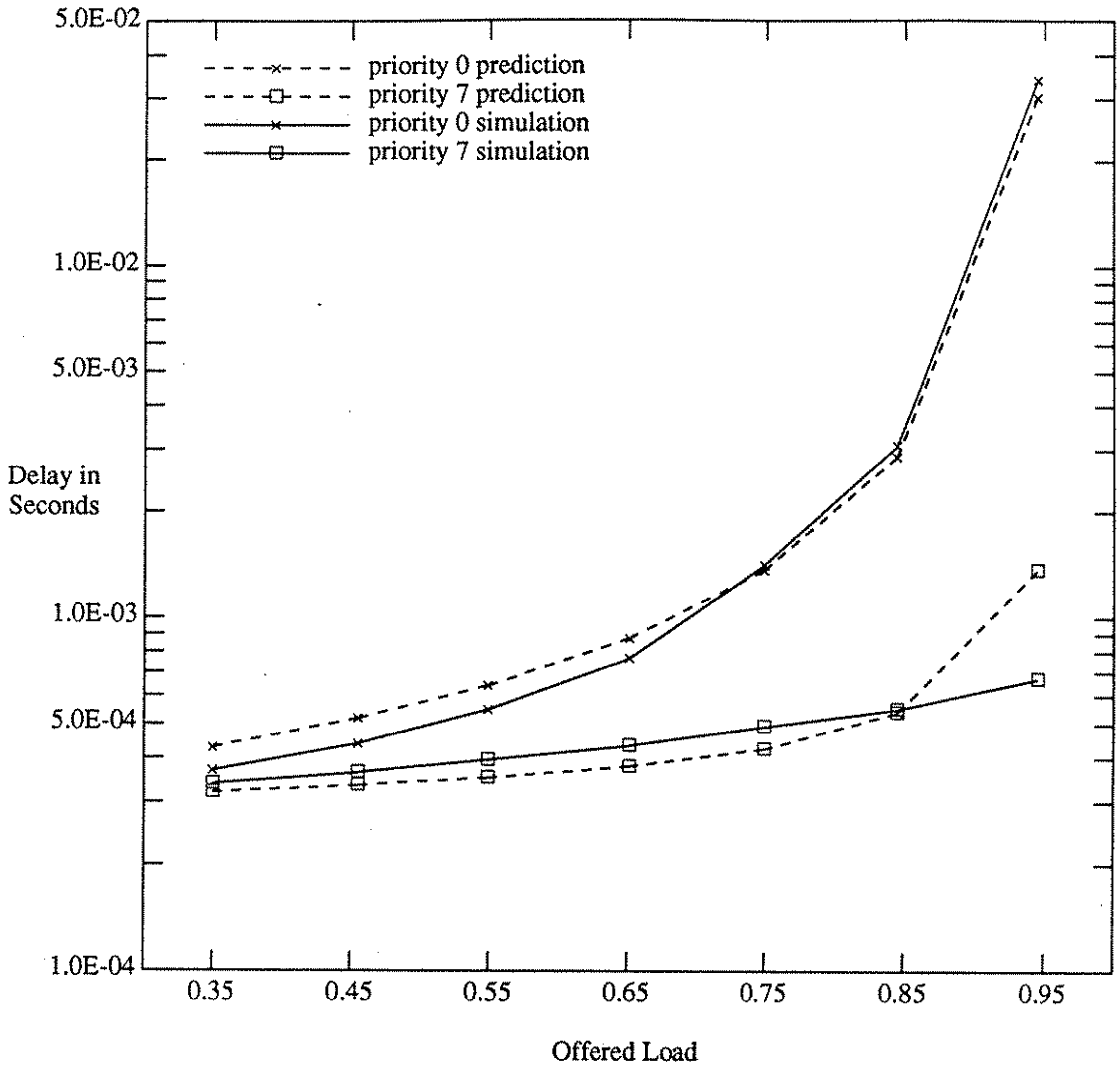


**Figure 2**
**Queueing Model vs. Simulation for 3 Stations**

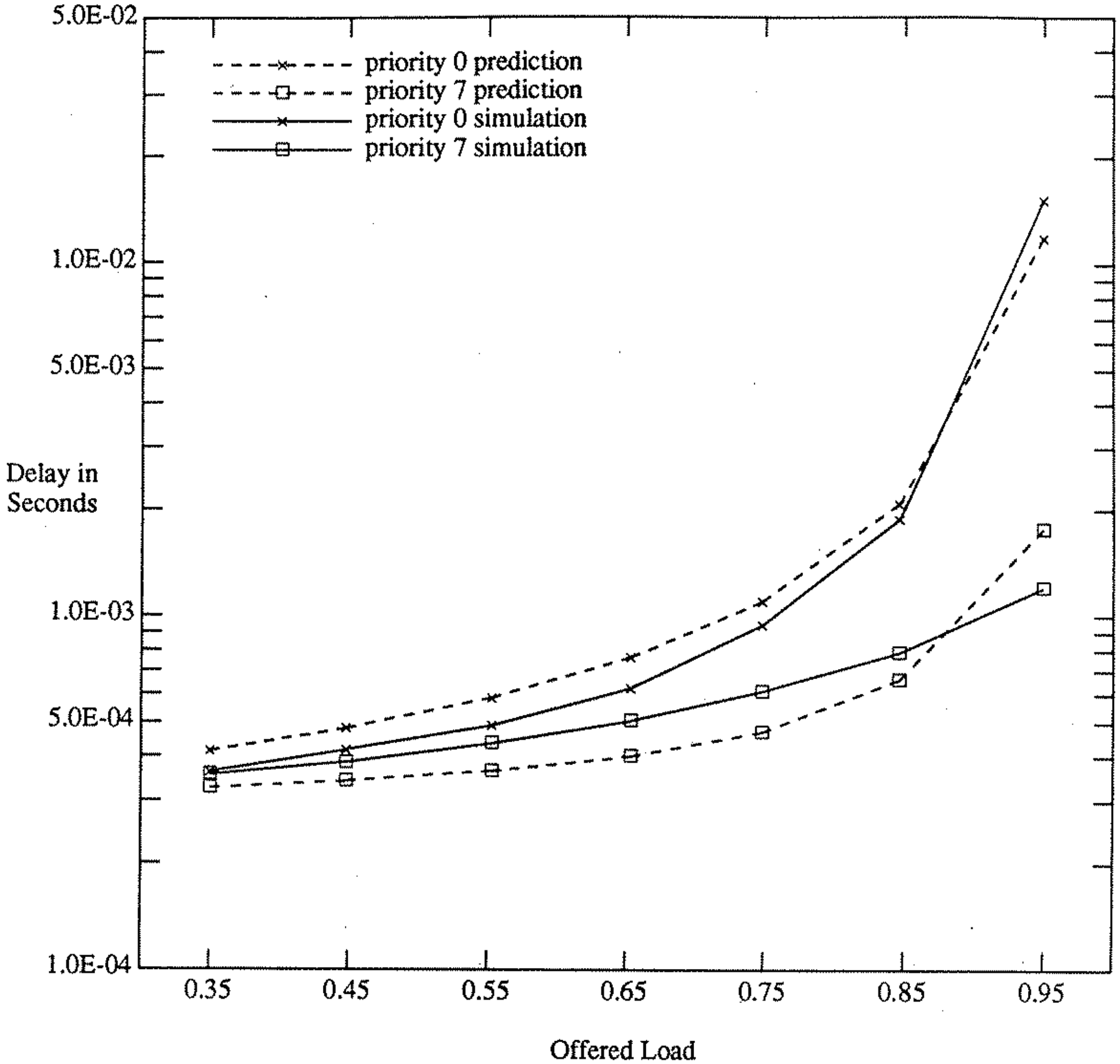


**Figure 3**
**Queueing Model vs. Simulation for 10 Stations**

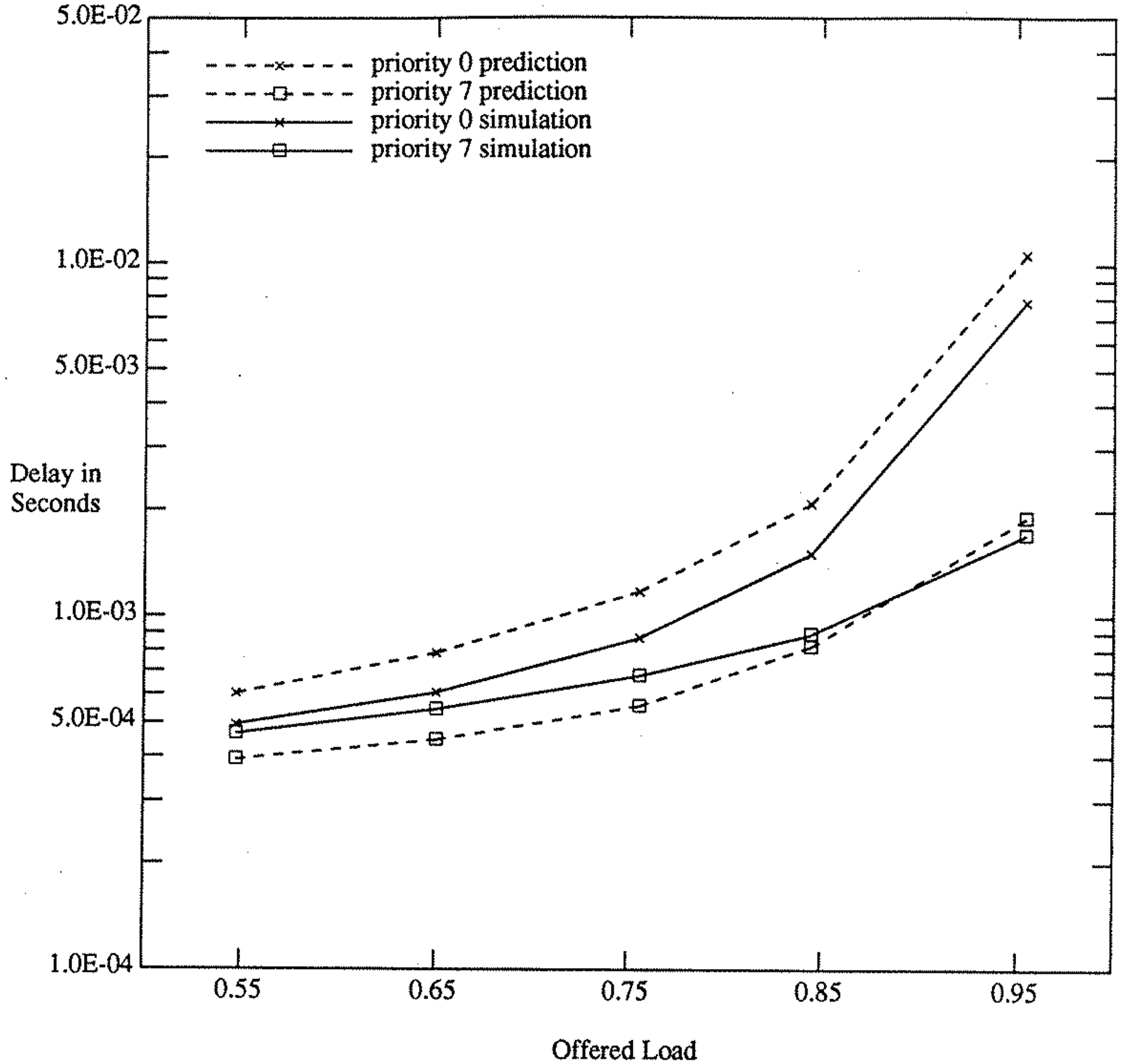


**Figure 4**
**Queueing Model vs. Simulation for 20 Stations**